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Subject

SPACE CHARGE LIMIT FOR A NEUTRALIZED BEAM

Following the notation of Laslett,* we have in a beam with neutralization η for the electric forces

$$K_E = -\frac{C}{B\beta^2}\left(1 + \frac{\epsilon_1 b(a+b)}{h^2}\right)(1-\eta),$$

for the dc magnetic forces

$$K_M = C\left(1 - \frac{\epsilon_2 b(a+b)}{g^2}\right),$$

for the ac magnetic forces

$$K_S = \left(\frac{1}{B} - 1\right)\left(1 + \frac{\epsilon_1 b(a+b)}{h^2}\right),$$

where C is an abbreviation for

$$C = \frac{2}{\pi}\,\frac{N r_p R}{\gamma b(a+b)} .$$

The shift in betatron frequency ν_y is given by

$$\Delta(\nu_y^2) = K_E + K_M + K_S .$$

Rearranging terms leads to the following expression for the space charge ν shift

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*L. J. Laslett, On Intensity Limitations by Transverse Space Charge Efforts in Circular Particle Accelerators, BNL-7534, p. 324 (1963).

$$\Delta\nu = \frac{Nr_p R}{\pi\gamma\nu} \left\{ \frac{\epsilon_1}{h^2} \left[1 + \frac{1}{B\beta^2}\left(\gamma^{-2} - \eta\right)\right] + \frac{\epsilon_2}{g^2} + \frac{1}{B\beta^2} \frac{(\gamma^{-2} - \eta)}{b\,(a+b)} \right\}.$$

This formula agrees with Laslett's for $\eta = 0$, and with the ISR design study (AR/Int. SG/64-9) for $B = 1$, the unbunched beam case, in the limit $\gamma >> 1$. In this limit which is the only one of importance for proton storage rings, we find:

$$\Delta\nu = \frac{Nr_p R}{\pi\gamma\nu} \left\{ \left(\frac{\epsilon_1}{h^2} + \frac{\epsilon_2}{g^2}\right) - \eta\left(\frac{\epsilon_1}{h^2} + \frac{1}{b\,(a+b)}\right) \right\}.$$

One may note that compensation of space charge forces by neutralization is not possible since $\epsilon_2/g^2 < 1/(b\,(a+b))$ in all practical cases. Numerical estimate for $\eta = 1$, a fully neutralized beam: $N = 10^{15}$, $R = 333$ m, $\gamma = 100$, $\nu = 18$, $h = 12.5$ mm, $g = 12.5$ mm, $a = 6.7$ mm, $b = 4.5$ mm, $\epsilon_1 = 0.2$, and $\epsilon_2 = 0.42$. a and b are circumferential averages assuming $\beta = 20$ m and $E_H = 2.2\,\pi$ μrad m,

$$E_V = 1.0\,\pi \ \mu\text{rad m}.$$

We find

$$\Delta\nu = 0.885\,(0.232 - 2.34\,\eta).$$

Here we assumed that magnets are only present over half the circumference and consequently took one half of the ϵ_2 term. Hence

for $\eta = 0$ $\qquad \Delta\nu = 0.205$

for $\eta = 1$ $\qquad \Delta\nu = -1.87$

Conclusions:

1) The unneutralized ν shift is close to the figure given in FN-168.

2) The neutralized ν shift is not tolerable, hence deneutralization is essential in the storage rings.